Quantum Fysica B

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2 opgaven, iedere uitwerking op een apart vel papier met naam en studie nummer
Maak gebruik van de bijgevoegde formulelijst waar dat nodig lijkt.

Opgave 1

The electron in a hydrogen atom occupies the combined position and spin state:

$$\Psi = R_{32}\left(i\sqrt{\frac{4}{5}} Y_2^{-1}\chi_- + \sqrt{\frac{1}{5}} Y_2^{-2}\chi_+ \right)$$

5 pnts a. Suppose that you can only measure the polar angles of the electron. What is the probability density for finding it at a certain (θ, ϕ)?

5 pnts b. What values might you get and with what probabilty if the following quantities are measured:

(a) S^2.
(b) S_z.
(c) J_z (where $\vec{J} = \vec{L} + \vec{S}$).
(d) J^2 (you do NOT have to give probabilities for J^2).

5 pnts c. Calculate the expectation value of:

(a) L_z.
(b) J^2.
(c) $S_x + iS_y$.
(d) r.

d. (Really difficult, only for bonus points, only if you have time left)
Suppose you were able to measure the y-component of the spin of the electron at any place in space. At what polar angles is the probability unity for measuring $S_y = +\frac{1}{2}\hbar$.

Opgave 2

The matrices representing S_x, S_y and S_z for a particle of spin 1 (in the basis χ_+, χ_0 and χ_- eigenstates of S_z) are:

$$S_x = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad S_y = \frac{i\hbar}{\sqrt{2}} \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \quad S_z = \hbar \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

5 pnts a. Write the conmutation rules that these operators should obey. Use one of them to obtain S_z from S_x and S_y.

b. Suppose that the particle is placed in a magnetic field of magnitude B_o which is parallel to the z-axis. At t=0 the the particle is the state $\begin{pmatrix} 1/2 \\ 1/\sqrt{2} \\ 1/2 \end{pmatrix}$.

5 pnts (a) Calculate the state of the system at a time $t > 0$ (the Hamiltonian in this case is given by $H = \gamma \vec{B} \cdot \vec{S}$).

5 pnts (b) Calculate t dependence of the expectation value of S_x.